PHYSICS

Projectile Motion
Physics Lab IV

Objective

The projectile motion experiment applies a working knowledge of kinematics for motion in two dimensional space. Students will utilize formulated equations to determine theoretical values of a projectile's position along a trajectory. Experimental data will be acquired and compared to the theoretical results.

Equipment List

Attention: You will need a pair of safety goggles while performing this experiment. If necessary the Physics Department will loan you a pair. Projectile Launcher, projectile, Smart Timer, Time-of-Flight Pad, 2 Table Clamps with rods, Wrench (optional), 1 and 2 Meter Sticks (horizontal rule and vertical rule), ruler, balsa wood window, Box Lid, Carbon Paper, Masking Tape, "Sticky" Putty, Scratch Paper, Scissors

Theoretical Background

In the Linear Kinematics experiment, the kinematic relations were examined for one-dimensional motion. Here the same relations will be extended to predict the motion of an object in two dimensions. The consequence of air resistance will be ignored in this experiment.

To begin the experiment, a steel ball will be launched from a spring-loaded gun. Once the ball has cleared the barrel of the gun, the force of gravity is responsible for accelerating the ball to the ground. Frictional forces are considered negligible. With this in mind, the motion of the ball can be subdivided into two types: motion in the horizontal direction and motion in the vertical direction. Note that the force of gravity acts on the ball only in the vertical direction. No external forces act on the ball in the horizontal direction, hence its horizontal acceleration is zero. The kinematic relations in the vertical and horizontal directions can be written in the following manner:

Horizontal Direction:

$$x = x_i + v_{ix}t \tag{1}$$

Vertical Direction:

$$v_{fy} = v_{iy} + at \tag{2}$$

$$y = y_i + v_{iy}t + \frac{1}{2}\,at^2 \tag{3}$$

$$v_{fy}{}^2 = v_{iy}{}^2 + 2a(y - y_i) \tag{4}$$

In these equations, x is the horizontal position of the ball, x_i is the initial horizontal position of the ball, v_{ix} is the initial velocity in the horizontal direction, t is the elapsed time, v_{fy} is the final velocity in the vertical direction, v_{iy} is the initial velocity in the y-direction, a is the acceleration in the y-direction, y_i is the initial height of the ball, and y is the vertical distance travelled (i.e. the height).

The initial velocities in the x and y direction are the vector components of the initial velocity. Figure 1 illustrates these relationships.
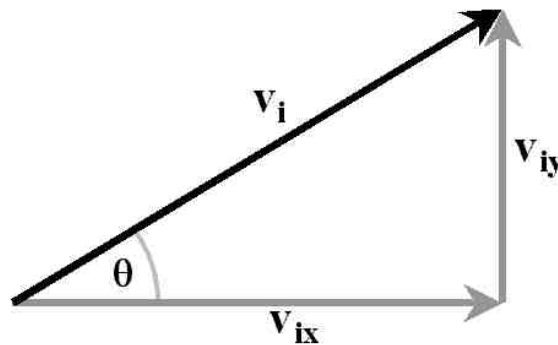

Figure 1: Vector components of the initial velocity

If the initial speed v_i of the ball and the launch angle, θ, are known, then the vector components can be found using methods of trigonometry:

$$v_{ix} = v_i \cos\theta, \tag{5}$$

$$v_{iy} = v_i \sin\theta. \tag{6}$$

Using these developed equations, the range, maximum height, and total time of flight will be calculated. These theoretical values will then be compared to experimental values.

Procedure and Data Analysis

Projectile Motion: Initial Velocity Measurement

Caution: You must wear your safety goggles at all times during this experiment. Failure to do so will find you expelled from the lab room. It is necessary to first determine the initial velocity of the ball when launched from the spring-loaded gun. This will be done by launching the ball horizontally, measuring the range and transit time, and calculating the initial velocity as the ratio of these quantities.

1. Tape a strip of carbon paper to the time-of-flight pad so that the carbon side faces toward the table top. Tape a small piece of scratch paper under the carbon paper so that, when the ball hits the pad, a small mark will be left on the paper.

2. Place some of the "sticky" putty on two corners of the time-of-flight pad to keep it from slipping on the table.

3. Attach the launcher to the rod at the end of the table, as shown in Figure 2. Set the initial height of the launcher to 50 cm and set the initial angle of the launcher to zero. Be sure the launch point, indicated on the launcher, is directly above the edge of the table.

Figure 2: Launcher attached to rod

4. Insert the ball into the launcher using the plunger. Set the launcher to the **Medium Range** mode. The spring "clicks" indicate the range mode. *Two* spring "clicks" indicates the **Medium Range** mode.

5. ***Before launching the ball, be sure all group members are two feet away from the table.***
   DO NOT LOOK INTO THE LAUNCHER BARREL WHEN THE LAUNCHER IS ARMED!!!!! Pull the cord on top of the launcher to launch the ball. Take care to notice the approximate location the ball strikes the table.

6. Position the time-of-flight pad so that the center of the pad is in the position that the ball impacts the table.

7. Insert the ball into the launcher and set the launcher to medium range mode again.

8. Turn on the Smart Timer and set it to **Time: Two Gates** mode. Press the **Start/Stop** key. An asterisk (*****) should appear on the screen of the Smart Timer.

9. Be sure all lab members are two feet away from the lab table. Pull the cord on top of the launcher to launch the ball. The total time of flight t_{total} should be on the Smart Timer, and a small mark should be on the paper on the Time-of-Flight pad indicating where the pad was struck. Record the total time of flight.

10. Repeats steps 7 through 9 for time of flight measurements t_{total} until 5 times are recorded.

11. Calculate the percent variation in the time of flight t_{total} and use this value as the percent uncertainty in the time of flight.

12. Measure the distance along the table from just below the launcher to the center of the marks on the Time-of-Flight pad. It might be useful to use the 2 m meter stick to measure to the edge of the Time-of-flight pad, then use a ruler to measure from the edge of the pad to the mark on the pad. Record this distance as the horizontal range of the ball, R.

13. Measure the distance between the two farthest impact points on the pad. Record half this distance as the uncertainty in the range R. Use this value, and the value of the range recorded previously to determine the percent uncertainty in the range.

14. Open the Excel program titled Projectile Motion Calculator from the computer's desktop. The program should open into the Medium Range Data spreadsheet. Enter the measured flight times and horizontal range. Record the calculated average time of flight and corrected initial velocity (corrected v_i) values on your data sheet.

Figure 3: Launcher attached to table clamp

Projectile Motion: Varying the Launch Angle

In this part of the experiment, the range, maximum height, and total transit time will be calculated, and confirmed through experimentation. Notice, in the first exercise the ball was fired from zero degrees. The Projectile Motion Calculator displayed a corrected velocity for an average flight time of the ball at zero degrees. In this exercise the calculator will correct for the ball's velocity when it is fired at angles greater than zero. **It is not necessary to re-enter the measured flight time or the range**.

1. Remove the paper from the pad, and tape a fresh piece to the pad.

2. Remove the launcher from the rod and attach it to the table clamp, as shown in Figure 3. The launcher should contain a wooden plank with a whole drilled into it. Fasten the launcher to the table clamp using a thumb-screw and hex-nut. Be sure the launcher's platform sits firmly beneath the table. Use the wrench to fix the position of the launcher.

3. Set the launcher so that the cross-hairs are level with the table and the plumb-bob hangs at the 45° mark. The initial velocity of the ball will be calculated using this reference point by the Projectile Motion Calculator.

   Recap: The Initial Velocity Measurement procedures were performed to establish a reference point at $x = 0$ cm, $y = 50$ cm and $\theta = 0°$. The Excel spreadsheet gives a corrected velocity determined from the angle of inclination. Therefore you only need to enter the launch angle of the projectile to obtain a corrected initial velocity. For example, type 45 for the value of theta and press enter. Record the displayed **corrected** initial velocity on your data table. **The lab instructor will provide assistance if necessary.**

4. Use the **corrected** initial velocity (corrected v_i) and the equations from the theoretical background to calculate the maximum height H, total time of flight of the ball t_{total},

the range of the ball R, and the horizontal position at which the height is maximum R_H. Each group member should perform this calculation. Show your work for this calculation on a separate sheet of paper. **The lab instructor will provide an example of this calculation.**

5. Group members will validate all calculation with each other before performing the experiment.

6. Use the Projectile Motion Calculator to check your calculations by typing the launch angle into the spreadsheet. The calculation results can be obtained by "clicking" the tab marked *Medium Range Theory Values* at the bottom of the spreadsheet.

7. After calculating H, t_{total}, R and R_H, set the table-clamp-and-rod assembly at the side of the table to your R_H value. Set the balsa wood window, which is also referred to as the height selector, to your H value. The middle of the time-of-flight pad should be placed at the distance R away from the launcher.

8. Using the procedure outlined for the previous exercise, set the launcher to the **Medium Range** mode. Set the Smart Timer to measure the total time of flight. Launch the ball. If the ball hits the height selector, or fails to hit the pad and leaves a mark on the paper, realign the height selector and pad, and re-launch the ball.

9. Measure and record the total time of flight, the range of the ball, and the maximum height (if the height selector had to be adjusted). Calculate the percent difference in each of the quantities. After measuring each range, place a "X" over the mark on the pad so as to discard that data point.(See item 10)

10. Repeat steps 3-9 for initial angles of $35°, 55°$ and $65°$. Each person at the table should perform the calculations for a different angle. Use the Excel spreadsheet to check you calculations.

Selected Questions

1. How does the range of the ball vary as the angle increases? From your data, what was the angle of maximum range?

2. Review your results from the "Varying the Launch Angle" exercise. (a) Is there a relationship between the maximum height of the ball and the total travel time? (b) Is there a relationship between the range of flight of the ball and the total travel time? Explain your reasoning for both parts.

3. Frictional forces were not accounted for during measurements of maximum range, total time of flight, and the maximum height of the ball. (a) What affect would friction have on these values compared to the calculated values (increase, decrease or no effect)? (b) Would friction be classified as a random or systematic error? Explain your reasoning for both parts.

4. A ball is fired by a cannon from the top of a cliff as shown below. Which of the paths 1-5 would the cannon ball most closely follow?
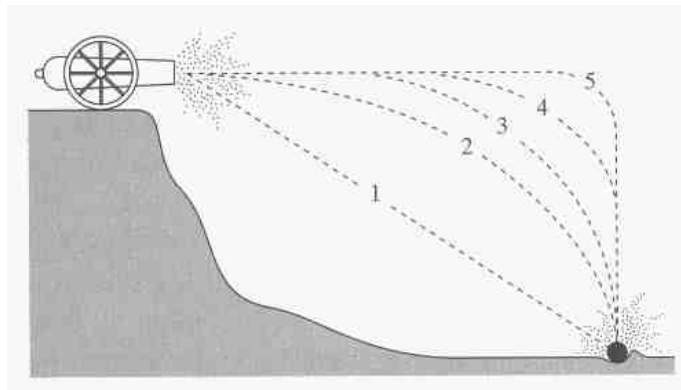
5. A battleship simultaneously fires two shells with the same initial velocity at enemy ships. If the shells follow the parabolic trajectories shown, which ship gets hit first? [You may want to look over your data from projectile motion on a level surface before answering this question]